

Series 01: Numerical series

Exercise 01 : Tell whether the following series converges, if yes, calculate their sum:

- $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
- $\sum_{n=1}^{\infty} 2\sqrt{n} - \sqrt{n+1} - \sqrt{n-1}$

Exercise 02 : Study the convergence of the series $\sum_{n=1}^{\infty} u_n$, in the following cases:

- $u_n = \sqrt{n^2 + n} - n$, $u_n = \left(\cos\left(\frac{1}{n}\right)\right)^{n^2}$, $u_n = e^{-n}$, $u_n = 1 - \cos\left(\frac{1}{\sqrt{n}}\right)$,
- $u_n = \frac{2^n + n^3}{3^n + n^2}$, $u_n = \ln\left(\frac{2 + n^\alpha}{1 + n^\alpha}\right)$, $\alpha \in \mathbb{R}$, $u_n = \sin\left(\frac{1}{n}\right)$, $u_n = \sqrt[n]{n} - 1$.

Exercise 03 : Using the Root test and the Ratio test, study the convergence of the following series:

- $\sum_{n=2}^{\infty} \frac{n}{(\ln(n))^n}$, $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$
- $\sum_{n=0}^{\infty} \frac{3^n}{n!}$, $\sum_{n=0}^{\infty} \frac{(n!)^3}{(3n!)}$.

Exercise 04 : Let

- $\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$,
- $\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{1+n}$.

Determine whether the series is absolutely convergent, conditionally convergent or divergent.