

Exercise 1:

A mass $m = 0.453\text{ kg}$ attached to a spring stretches it by 7.787 mm at rest. Determine the angular frequency of the mass-spring system.

Exercise 2:

A harmonic oscillator consisting of a mass $m = 2\text{ kg}$ and a spring with stiffness constant k has an angular frequency of $\omega_0 = 10\text{ rads/s}$ and a mechanical energy of $E_m = 5\text{ Joules}$. Calculate the amplitude of the oscillations of the system.

Exercise 3:

A solid of mass m that can slide without friction on a horizontal surface is fixed to a spring with stiffness $k = 48\text{ N/m}$. Its elongation x measured from its equilibrium position is given by $x = x_m \sin(8t - \pi)$ to make the mass m oscillate, it is given an energy of 0.24 J . Determine:

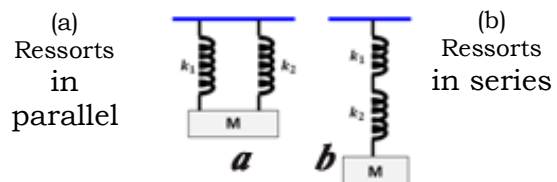
- The mass m of the solid.
- The amplitude of the motion.
- The maximum velocity of the oscillator.
- The elongation of the oscillator for which the potential energy is equal to half of the kinetic energy.
- The components of the velocity and acceleration at this point.

Exercise 4:

The ratio k/m of a vertical mass-spring system is equal to 4. If at $t = 0$ the mass is pulled down 2 cm from its equilibrium position and released with a velocity of 8 cm/s , determine the resulting motion and the maximum acceleration.

Exercise 5:

Determine the equivalent stiffness constants, k_{eq} , of the following systems (Fig.1):
 Compare k_{eq} with the equivalent capacitances, C_{eq} , of two capacitors in series and in parallel



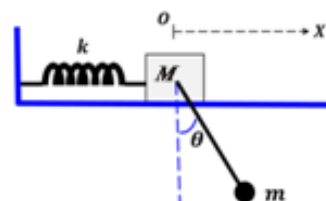
(Fig.1)

Exercise 6:

A mass-spring system m_1, k_1 , has a natural angular frequency ω_1 . a second spring with stiffness k_2 is added in series with the first spring, the natural frequency of the system is $\frac{1}{2}\omega_1$. Determine k_2 in terms of k_1 .

Exercise 7:

A simple pendulum of length l and mass m , attached to a mass M that can slide without friction on a horizontal plane as shown in figure 2, oscillates in the vertical plane. Let x be the displacement of M and θ be the angle of rotation of the pendulum. Write the Lagrangian of the system.



(Fig.2)

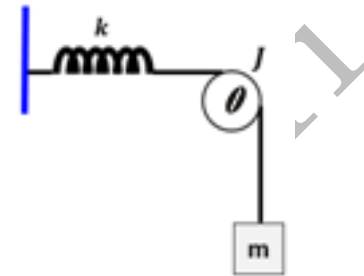
Exercise 8:

Consider the mechanical system shown in figure 3 below. The cylinder of radius R , mass m , and moment of inertia j can roll without slipping on the horizontal plane. Determine the frequency of the oscillatory motion of the cylinder

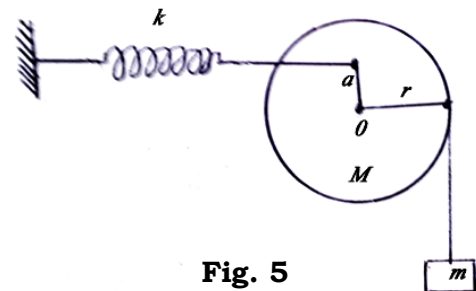
**(Fig.3)****Exercise 9:**

Consider the mechanical system shown in figure 4. The disk of moment of inertia J , radius R , and mass M can rotate freely around the horizontal axis passing through O . The rope supporting the mass m is inextensible and does not slip on the disk.

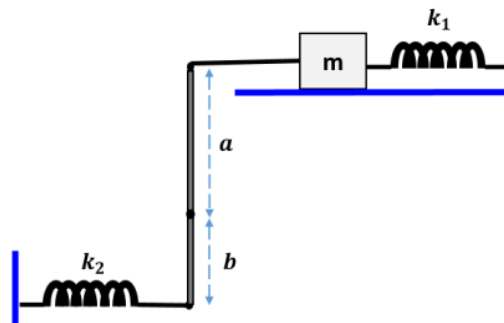
Determine the equation of motion of m knowing that at the initial time the mass m is pulled down 4 cm from its equilibrium position and released without velocity. Calculate the angular frequency of the oscillations.

**(Fig.4)****Exercise 10:**

In the following figure (figure 5), a spring with coefficient k from one side attached to a disc at distance a from O and from other side is attached to the wall. This disc can rotate around point O . A mass with a non-deformable cable is connected to this disc. Find the equivalent stiffness and natural frequency for this structure. Choose the angular displacement θ of the disc as the generalized coordinate.

**Fig. 5****Exercise 11:**

Determine the angular frequency of oscillations of the system shown in figure 6 in the case of small movements.

**(Fig.6)**

Exercise 12:

A pinion (radius r , polar moment of inertia J) (Fig.7) is fixed to a shaft of torsional stiffness k_t and rolls without slipping on a rack of mass m , which is connected to a spring of stiffness k . Using $x = r\theta$ as the generalized coordinate, derive the equation of motion via the Lagrangian method and find the natural angular frequency.

