Series 03

Exercice 01

1. Let the paths:
$$\gamma_1 : [1, i], \quad \gamma_2 : [1, 1+i, i], \quad \gamma_3 : [1, 1+i], \quad \gamma_4 : [1+i, i].$$
   (a) Calculate the following line integrals:
   - 1) $\int_{\gamma_1} \overline{z}\, dz$, 2) $\int_{\gamma_2} \overline{z}\, dz$,
   - 3) $\int_{\gamma_1} z\, dz$, 4) $\int_{\gamma_2} z\, dz$.

   (b) Conclude.

2. Calculate
$$\int_{\gamma} z \sin(z)\, dz$$
such that γ is the straight line segment joining $z_0 = 0$ to $z_1 = i$.

Exercice 02
Calculate the following line integrals:

1. $\int_{\gamma}(z^2 + 3z)\, dz$ if
   - γ is the straight line segment joining $z_0 : (0, 0)$ to $z_1 : (0, 1)$.
   - γ is the quarter of the circle of center $(0, 0)$ joining $z_0 : (2, 0)$ to $z_1 : (0, -2)$.
2. $\int_{\gamma}(2\overline{z} + 3|z|^2)\, dz$ if
   - γ is the half of the circle of center $(0, 0)$ joining $z_0 : (2, 0)$ to $z_1 : (-2, 0)$.
3. $\int_{\gamma} 2i\overline{z}\, dz$ if
   - γ is the path $|z - i| = 2$.

Exercice 03
Using Cauchy theorem and Cauchy integral formula, calculate the following line integrals:

1. $\int_{|z|=1} \frac{e^{z^2}}{(z-2i)^2}\, dz$
2. $\int_{|z-i|=2} \frac{\cos(\pi z)}{(z+2i)^2(z-2i)^2}\, dz$
3. $\int_{|z|=3} \frac{e^z}{z^2(z^2-2z+2)}\, dz$