Series 01

Exercise 01:

1. Write in the standard form (algebraic form) the following complex numbers:
   - $z = \dfrac{(1+i)^9}{(1-i)^7}$,
   - $z = \dfrac{1+\alpha i}{2\alpha + (\alpha^2-1)i}, \quad \alpha \in \mathbb{R}.$
2. Write in the trigonometric and exponential form the complex number:
   - $z = \dfrac{1+i\sqrt{3}}{1-i}$.

Exercise 02: Represent geometrically the following sets in the complex plane:

1. $\{z \in \mathbb{C} \mid |z-3i| \leq |z-3|\}$,
2. $\{z \in \mathbb{C} \mid |z-i| < 3\}$,
3. $\{z \in \mathbb{C} \mid \text{Re}(z) - \text{Im}(z) < 1\}$.

Exercise 03: Let the complex function $f : \mathbb{C} \to \mathbb{C}$ defined by

$$f(z) = \frac{z-2}{z+i}.$$

- Determine the set of points for which $f(z) \in \mathbb{R}$,
- Determine the set of points for which $f(z) \in i\mathbb{R}$.

Exercise 04: Compute the n^{th} root of the following complex numbers, with different values of n

$$\sqrt{i}, \quad \sqrt[3]{i}, \quad \sqrt[5]{-1}, \quad \sqrt[5]{1}, \quad \sqrt[6]{-8}$$

Exercise 05: (Supplementary exercise)

1. Let $z = e^{i\theta/2}$, prove that:

$$\frac{\sin(\theta)}{2} + \frac{\sin(2\theta)}{2^2} + \frac{\sin(3\theta)}{2^3} + \cdots = \frac{2\sin(\theta)}{5-4\cos(\theta)}.$$

2. Let $z, z' \in \mathbb{C}$ such that

$$|z| = |z'| = 1, \quad 1 + zz' \neq 0, \quad W = \frac{z+z'}{1+zz'}.$$

   - Show that $\overline{z} = \dfrac{1}{z}$,
   - Calculate W,
   - What do you conclude?