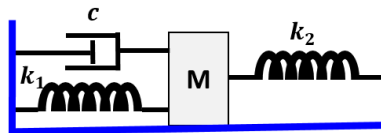


### Exercise 1:

Consider the following mechanical system:



- Write the equation of motion for the mass  $M$  along the  $x$  -axis.
- For what values of  $c$  is the system's motion damped oscillatory?
- Determine the general expression for  $x(t)$  for  $c = 0,1 \text{ Nm}^{-1}\text{s}$ , the logarithmic decrement  $\delta$  and the quality factor  $Q$  of the system.  
 $M = 1 \text{ kg}$ ,  $k_1 = 20 \text{ N/m}$ ,  $k_2 = 5 \text{ N/m}$

### Exercise 2:

A body of mass  $m = 1 \text{ kg}$  is suspended from a fixed point by a spring with a stiffness constant  $k$ , immersed in a liquid. The liquid exerts a frictional force on the body equal to  $-c\vec{v}$  where  $\vec{v}$ , is the velocity of the body and  $c$  is a constant equal to  $0,4 \text{ Nm}^{-1}\text{s}$ .

- Determine the values of  $k$  for which the motion of the body in the liquid is damped oscillatory.
- If the motion of the body is damped oscillatory with a logarithmic decrement  $\delta = 10^{-2}$ , determine the pseudo-period and the stiffness constant of the spring.

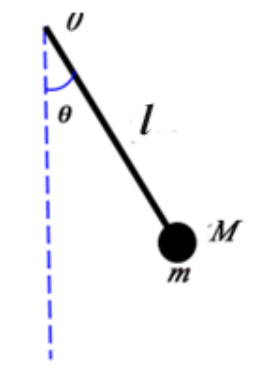
### Exercise 3:

A block of mass  $m = 7 \text{ kg}$  is suspended from a spring with a stiffness  $k = 600 \text{ N/m}$ . If the block receives an upward velocity  $v = 0.6 \text{ m/s}$  from its equilibrium position at  $t = 0$ , determine its position as a function of time  $x(t)$ . Assume that its positive displacement is downward, and its motion occurs in a medium that provides a damping force  $F = (50 |v|) \text{ N}$ , where  $v$  is in  $\text{m/s}$ .

### Exercise 4:

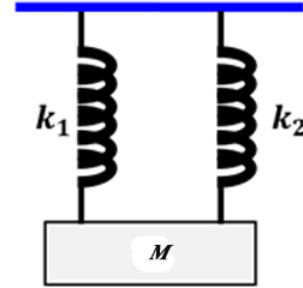
A rod  $OM$ , of length  $l$ , negligible mass, carries a mass  $m$  considered as a point mass at its end. It is movable in the vertical plane, and the articulation at  $O$  is perfect.

- Write the equation of motion for  $m$  for small amplitude oscillations. Write the general expression for  $\theta(t)$
- What happens to this equation if the rod is subjected to an external force perpendicular to the rod and of magnitude  $F$  at a point  $A$  located at a distance  $OA = a$  from the suspension point  $O$ .
- What happens to this equation if the mass is subjected to viscous frictional force given by the equation  $\vec{F}_f = -c\vec{v}$ , where  $c$  is a positive constant and  $\vec{v}$  is the velocity of the mass. Determine the frequency of damped oscillations in the case of weak damping



**Exercise 5:**

A mass  $m = 10\text{kg}$  is suspended from a ceiling via two springs with stiffness  $k_1 = 30\text{N/m}$ ,  $k_2 = 70\text{N/m}$  as shown in the figure. The mass  $m$  is subjected to a force  $F(t)$  directed along  $x$  with expression  $F(t) = 0,2\sin 3t$ .



- Write the equation of motion for the mass.
- Determine the displacement  $x$  of the mass as a function of time  $t$  if at  $t = 0$ ,  $x = 5\text{cm}$  and  $\dot{x} = 0$

**Exercise 6:**

A body of mass  $m = 1\text{kg}$  is connected to a fixed point  $A$  by a spring with a stiffness constant  $k = 9 \times 10^2\text{N/m}$ . The body can slide without friction on the horizontal plane.

- Determine the natural frequency of the system's oscillations.
- Determine  $x(t)$  knowing that at  $t = 0\text{s}$ ,  $x = 4\text{cm}$  and  $\dot{x} = 0\text{m/s}$ .

In a viscous fluid where the frictional force is of the form  $\vec{F}_f = -c\vec{v}$ , the system undergoes damped oscillations with a pseudo-frequency  $\omega_a = 5,45\text{rad/s}$ .

- Deduce the friction coefficient  $c$ .
- Calculate the logarithmic decrement.
- Calculate the quality factor  $Q$  of the system.

Calculate  $x(t)$  at  $t = 100T_a$  if at  $t = 0\text{s}$ ,  $x = 2\text{cm}$ , where  $T_a$  is the pseudo-period of the motion.

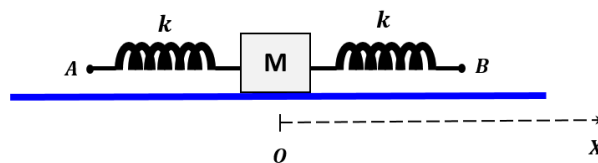
**Exercise 7:**

The ratio of successive amplitudes of a single-degree-of-freedom damped system is  $18/1$ . Determine the ratio of successive amplitudes if the damping ratio  $\zeta$  is:

- doubled
- halved.

**Exercise 8:**

Consider the system shown in the figure below:



Determine the natural frequency of the oscillations of the mass  $m$  if points  $A$  and  $B$  are fixed.

If points  $A$  and  $B$  undergo vibrations in the  $x$  direction of the form  $-C\cos\omega_1 t$  et  $C\cos\omega_2 t$ , respectively:

- Determine  $x(t)$ .
- What are the values of the resonance frequencies?