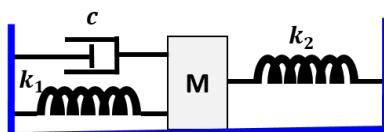


Exercise 1:

Consider the following mechanical system:



- Write the equation of motion for the mass M along the x -axis.
- For what values of c is the system's motion damped oscillatory?
- Determine the general expression for $x(t)$ for $c = 0,1 \text{ Nm}^{-1}\text{s}$, the logarithmic decrement δ and the quality factor Q of the system.

$$M = 1\text{ kg}, k_1 = 20\text{ N/m}, k_2 = 5\text{ N/m}$$

Exercise 2:

A body of mass $m = 1\text{ kg}$ is suspended from a fixed point by a spring with a stiffness k , immersed in a liquid. The liquid exerts a frictional force on the body equal to $-c\vec{v}$ where \vec{v} is the velocity of the body and c is a constant equal to $0,4 \text{ Nm}^{-1}\text{s}$.

- Determine the values of k for which the motion of the body in the liquid is damped oscillatory.
- If the motion of the body is damped oscillatory with a logarithmic decrement $\delta = 10^{-2}$, determine the pseudo-period and the stiffness constant of the spring.

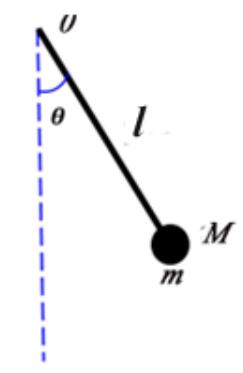
Exercise 3:

A block of mass $m = 7\text{ kg}$ is suspended from a spring with a stiffness $k = 600 \text{ N/m}$. If the block receives an upward velocity $v = 0.6\text{ m/s}$ from its equilibrium position at $t = 0$, determine its position as a function of time $x(t)$. Assume that its positive displacement is downward, and its motion occurs in a medium that provides a damping force $F = (50 |v|) \text{ N}$, where v is in m/s.

Exercise 4:

A rod OM , of length l , negligible mass, carries a mass m considered as a point mass at its end. It is movable in the vertical plane, and the articulation at O is perfect.

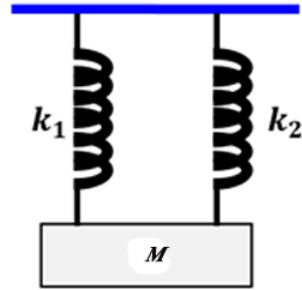
- Write the equation of motion for m for small amplitude oscillations. Write the general expression for $\theta(t)$
- What happens to this equation if the rod is subjected to an external force perpendicular to the rod and of magnitude F at a point A located at a distance $OA = a$ from the suspension point O.
- What happens to this equation if the mass is subjected to viscous frictional force given by the equation $\vec{F}_f = -c\vec{v}$, where c is a positive constant and \vec{v} is the velocity of the mass. Determine the frequency of damped oscillations in the case of weak damping



Exercise 5:

A mass $m = 10\text{kg}$ is suspended from a ceiling via two springs with stiffness $k_1 = 30\text{N/m}$, $k_2 = 70\text{N/m}$ as shown in the figure. The mass m is subjected to a force $F(t)$ directed along x with expression $F(t) = 0,2\sin 3t$.

- Write the equation of motion for the mass.
- Determine the displacement x of the mass as a function of time t if at $t = 0$, $x = 5\text{cm}$ and $\dot{x} = 0$

**Exercise 6:**

A body of mass $m = 1\text{kg}$ is connected to a fixed point A by a spring with a stiffness constant $k = 9 \times 10^2 \text{N/m}$. The body can slide without friction on the horizontal plane.

- Determine the natural frequency of the system's oscillations.
- Determine $x(t)$ knowing that at $t = 0\text{s}$, $x = 4\text{cm}$ and $\dot{x} = 0\text{m/s}$.

In a viscous fluid where the frictional force is of the form $\vec{F}_f = -c\vec{v}$, the system undergoes damped oscillations with a pseudo-frequency $\omega_a = 5,45\text{rad/s}$.

- Deduce the friction coefficient c .
- Calculate the logarithmic decrement.
- Calculate the quality factor Q of the system.

Calculate $x(t)$ at $t = 100T_a$ if at $t = 0\text{s}$, $x = 2\text{cm}$, where T_a is the pseudo-period of the motion.

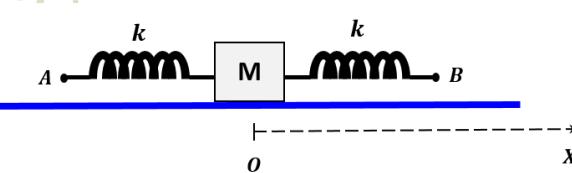
Exercise 7:

The ratio of successive amplitudes of a single-degree-of-freedom damped system is 18/1. Determine the ratio of successive amplitudes if the damping ratio ζ is:

- doubled
- halved.

Exercise 8:

Consider the system shown in the figure below:



Determine the natural frequency of the oscillations of the mass m if points A and B are fixed.

If points A and B undergo vibrations in the x direction of the form $-C\cos\omega_1 t$ et $C\cos\omega_2 t$, respectively:

- Determine $x(t)$.
- What are the values of the resonance frequencies?