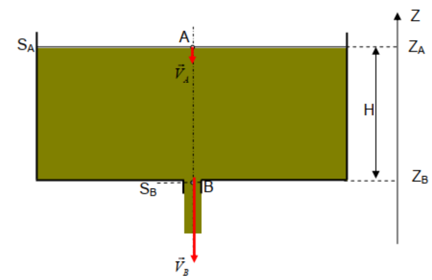
Exercise 1

The cylindrical reservoir (See figure), open to the atmosphere, has a cross-section SA with a diameter DA = 2 m. At its base, it features a drainage orifice with a cross-section SB and a diameter DB = 14 mm. The reservoir is filled to a height H=(ZA –ZB)= 2,5 m, with fuel oil, which is assumed to behave as an ideal fluid, possessing a density ρ=817 kg/m^3 .

The following data is provided:

- Atmospheric pressure Patm= 1 bar.
- Gravitational acceleration g=9,8 m/s^2.
- The ratio α=(SB/SA)



Part 1: The orifice is closed with a plug.

By applying the fundamental relationship of hydrostatics, determine the pressure PB at point B.

From this, deduce the value of the pressure force FB exerted on the plug.

Part 2: The orifice is opened. We proceed to drain the reservoir. The fuel oil flows out of the reservoir. Its average outflow velocity at point A is denoted as VA, and the outflow velocity at the orifice is denoted as VB.

1) Write the continuity equation. From this, derive VA in terms of VB and α.
2) By applying Bernoulli's theorem between points A and B, derive an expression for the velocity VB as a function of (g), (H), and (α).
3) Calculate the value of α. Is the assumption that the fluid level H remains constant valid? Justify your response.
4) Calculate VB under the assumption that α<<1.
5) Determine the volumetric flow rate Q_V of the fluid flowing through the orifice (in liters per second).
6) What would be the duration T of the drainage if this flow rate remained constant?

Exercise 2

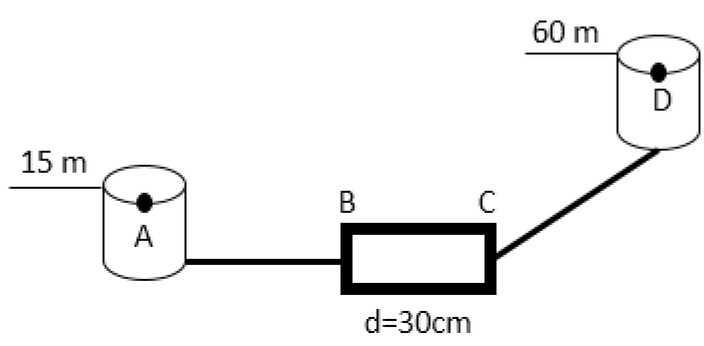
Calculate the head loss for a 1000 meter length of cast iron pipe with an internal diameter of 30 cm and a roughness of 0.24 mm, through which water flows at a velocity of 1.5 m/s.

Exercise 3

In the system depicted in the following figure, pump BC is required to transport petroleum oil with a flow rate of 160 l/s and a density of d = 0.762 to reservoir D.

Assuming that the energy loss from A to B is 2.5 m and the energy loss between C and D is 6.5 m.

- Calculate the energy that the pump must provide to the system



Solution 1

Part 1 :

1) $P_B = P_A + \rho g H$
   $= 10^5 + 817 \times 9.81 \times 2.5 \equiv 1.2 \times 10^5\ Pa$
2) $F_B = P_B \times S_B = P_B.\frac{\pi D^2}{4} = 1.2 \times 10^5.\frac{\pi(14.10^{-3})^2}{4} = 18.472\ N$

Part 2 :

1) Continuity equation $S_A.V_A = S_B.V_B \rightarrow V_A = \alpha V_B$
2) Bernoulli's eq : $\frac{V_B^2 - V_A^2}{2g} + \frac{P_B - P_A}{\rho g} + (Z_B - Z_A) = 0$
   With $P_A = P_B = Patm$, $(Z_B - Z_A) = H, V_A = \alpha V_B$
   $$V_B = \sqrt{\frac{2gH}{1-\alpha^2}}$$
3) $\alpha = \frac{S_B}{S_A} = \left(\frac{D_B}{D_A}\right)^2 = \left(\frac{14.10^{-3}}{2}\right)^2 = 4.9.10^{-5}$ The hypothesis of considering a quasi-constant level is right because; alpha t$\ll$ 1 donc $V_A = 0$
4) $V_B = \sqrt{2gH} = 7m/s$
5) $Q_v = S_B V_B = \frac{\pi(14.10^{-3})^2}{4}.7 = 1.10^{-3}$ m^3/s = 1L/s
6) $T = \frac{Volume}{Q_v} = \frac{S.H}{Q_v} = \frac{\pi 2^2}{4.10^3} \times 2.5 = 7854\ s = 130mn = 2h10mn$

Solution 2

- Flow regime verification: Calculation of Reynolds

  Re=$\frac{Velocity \times Diameter}{kinematic\ viscosity} = \frac{1.5 \times 0.3}{10^{-6}} = 45 \times 10^4 > 10^5\ turbulent\ flow\ regime\ with\ roughness$

- The coefficient λ can be calculated by colbrook-white eq :
  $\frac{1}{\sqrt{\lambda}}$=-2log$\left[\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{\lambda}}\right]$ or directly by Achour et al $\frac{1}{\sqrt{\lambda}}$=-2log$\left[\frac{\varepsilon/D}{3.7} + \frac{4.5}{Re} log\frac{Re}{6.97}\right]$

So : $\lambda = 0.0195$

Hpc=$\frac{\lambda.L.V^2}{D.2g} = \frac{0.0195 \times 1000 \times 1.5^2}{0.3 \times 20} = 7.32m$

Solution 3

Bernoulli's eq between both reservoirs is : $\frac{V_A^2 - V_D^2}{2g} + \frac{P_A - P_D}{\rho g} + (Z_A - Z_D) + Hp - \sum PDC\ (A-D) = 0$

Velocity and pressure in both reservoirs = 0

$$Hp = \sum PDC\ (A-D) + (Z_D - Z_A) = 60 - 15 + (2.5 - 6.5) = 54m$$

The pump power in (W):Pn=$Hp \times Q \times \rho g$=54$\times 160 \times 10^{-3} \times 1000 \times 0.762 \times 9.81 = 64585.9\ W$