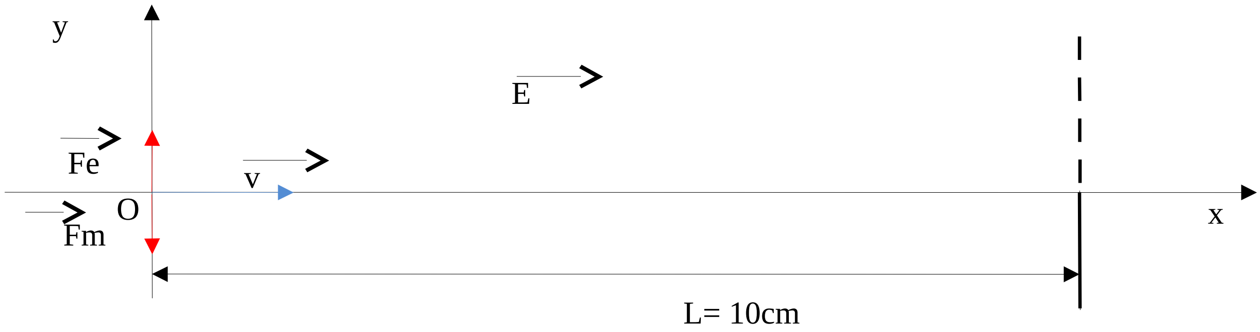
1st Year : Structure of Matter **Series N°2 : Main constituents of matter**

Exercise 1:

1. Determination of electron velocity:



The deviation y_0 cancels out when the electrostatic force and the magnetic force are opposite and equal.

$$F_e = F_m$$

$$eE = evB \Rightarrow v = \frac{E}{B}\ A.N: v = \frac{2.10^4}{10^{-3}} = 2.10^7\ m.s^{-1}$$

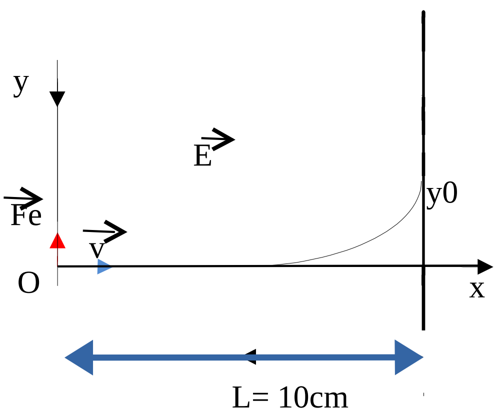
2. Kinetic energy of electrons:

$$E_C = \frac{1}{2} m_e v^2 = \frac{1}{2} 9,1.10^{-31}\left(2.10^7\right)^2 = 18,2.10^{-17}\ Joules$$

3. **Expression of electron beam y_0 deviation at capacitor output:**

From point O, the electron is deflected towards the positive plate of the capacitor under the action of an electric field E of intensity:

$$F_e = m\gamma \Rightarrow eE = m\gamma \Rightarrow \gamma = \frac{e}{m} E$$



The movement of electrons between the capacitor plates is:

- Uniform line following the Ox axis: $F_e = \gamma m \Rightarrow 0 = \gamma m \Rightarrow \gamma = 0$

$\gamma_x = 0$

$v_x = v_{0x} = cte$

$x = v_{0x} t + x_0 \Rightarrow x = vt \Rightarrow t = \frac{x}{v}$

- Rectilinear uniformly varied according to axis Oy : $F_e = m\gamma \Rightarrow q.E = m\gamma_y \Rightarrow \gamma_y = \frac{q.E}{m} = Cte$

$\gamma_y = Cte$

$v_y = \gamma_y t + v_{0y} \Rightarrow v_y = \gamma_y t$

$y = \frac{1}{2}\gamma_y t^2 + v_{0y} t + y_0 \Rightarrow y = \frac{1}{2}\gamma t^2$

The trajectory equation is obtained by replacing t and γ :

$$y = \frac{1}{2}\gamma t^2 \Rightarrow y = \frac{1}{2}\frac{e}{m}\frac{E}{v^2}x^2$$

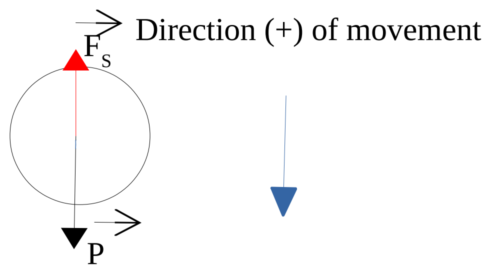
At capacitor output y = y_0, x = L $\Rightarrow$ $y_0 = \frac{1}{2}\frac{e}{m_e}\frac{E}{v^2}L^2$

A.N : $y_0 = \frac{1}{2} \times 1{,}76\,10^{11} \frac{2\,10^4}{(2.10^7)^2} 10^{-2} = 4{,}4\,10^{-2} m = 4{,}4\,cm$.

Exercise 2:

1- In the absence of the electric field the droplet suffered a free fall of the movement is descending (The thrust of Archimedes is negligible).

The balance of forces is written: $\sum \vec{F} = \vec{0} \Rightarrow \vec{P} + \vec{F_S} = \vec{0}$



$P - F_s = 0 \Rightarrow mg - 6\pi\eta r v_1 = 0 \Rightarrow mg = 6\pi\eta r v_1$

$We\,have: \rho = \frac{m}{V} \Rightarrow m = \rho V = \rho \frac{4}{3}\pi r^3$

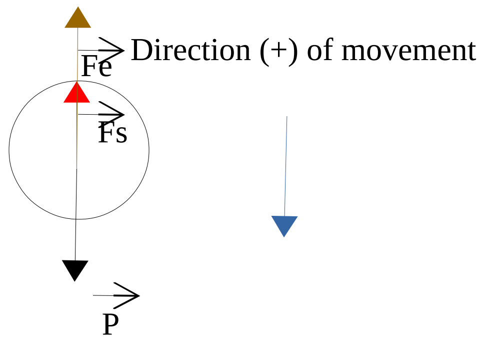
$\rho \frac{4}{3}\pi r^3 g = 6\pi\eta r v_1 \Rightarrow r^2 = \frac{3 \times 6\eta v_1}{4\rho g} \Rightarrow r = 3\sqrt{\frac{\eta v_1}{2\rho g}}$

A.N: $v_1 = \frac{d}{t} = \frac{2{,}25.10^{-3}}{10} = 2{,}25.10^{-4} m.s^{-1}$

$\rho = 0.885\,g.cm^{-3} = 885\,Kg.m^3 \Rightarrow r = 3\sqrt{\frac{1{,}80.10^{-5} \times 2{,}25.}{2 \times 885 \times 9{,}81}} = 1{,}45.10^{-6} m$

$$m=\rho V=\rho\frac{4}{3}\pi r^3=885\times\frac{4}{3}\times 3{,}14\times(1{,}45.10^{-6})^3=11{,}29.10^{-15}\,Kg$$

2- The droplet is now moving upwards with a velocity v_2:



Under the effect of the electric field $E=\frac{U}{d}=\frac{600}{6\times 10^{-3}}=10^5\,V.m^{-1}$

The balance of forces is written:

$$\vec{P}+\vec{F_S}+\vec{F_e}=\vec{0}\Longrightarrow P-F_S-F_e=0$$

$$mg-6\pi r\eta v_2-qE=0\Longrightarrow mg-6\pi r\eta v_2=qE\Longrightarrow q=\frac{mg-6\pi r\eta v_2}{E}$$

$$q=\frac{11{,}29.10^{-15}\times 9{,}81-6\times 3{,}14\times 1{,}80\,10^{-5}\times 1{,}45\,10^{-6}\times 1{,}27.10^{-4}}{10^5}=4{,}83\,10^{-19}\,C$$

$$q=n.e\Longrightarrow n=\frac{q}{e}=\frac{4{,}83\,10^{-19}}{1{,}6\,10^{-19}}.=3{,}01\approx 3$$

So the charge $q=3.e$

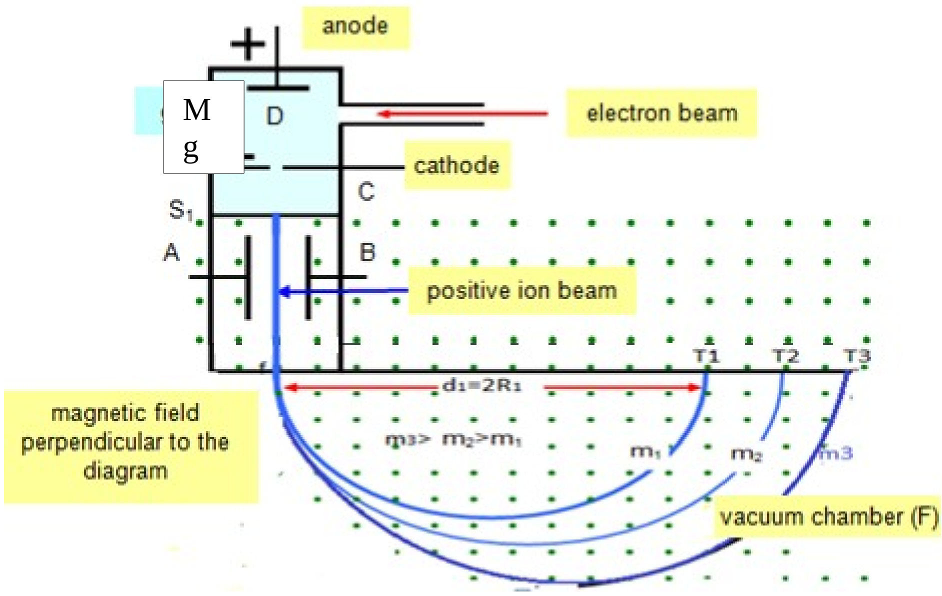
Exercise 3:

1- number of isotopes of natural magnesium is **3 isotopes**

Since the ion path in the analyzer is circular, one can write:

$$\vec{F_m}=\vec{F_c}$$

$$qvB=m\frac{v^2}{R}\Longrightarrow\frac{qB}{v}=\frac{m}{R}$$



1 - The lightest ion corresponds to the smallest diameter (d_1 = 2.5cm)

The ions $^{A1}Mg^{+2}$ of mass m_1 (A_1) and load q = 2e, describe a radius circumference R_1:

$$R_1=\frac{m_1}{q}\frac{v}{B}=\frac{d_1}{2}$$

$$m_1=\frac{qBd_1}{2v}=\frac{2\times 1{,}6.10^{-19}\times 1\times 2{,}5.10^{-2}}{2\times 10^5}=4.10^{-26}\,Kg$$

$$m_1=\frac{40.10^{-27}}{1{,}66.10^{-27}}=24{,}096\,a.m.u$$

2 - The $^{A2}Mg^{+2}$ ions of atomic mass m_2 (A_2) and charge q = 2e describe a radius circumference R_2:

$$R_2=\frac{m_2}{q}\frac{v}{B}=\frac{d_2}{2}$$

The $^{A3}Mg^{+2}$ ions of mass m_3 (A_3) and load q = 2e describe a radius circumference R_3:

$$R_3=\frac{m_3}{q}\frac{v}{B}=\frac{d_3}{2}$$

$$\frac{qB}{v}=\frac{m}{R}\;\frac{m_1}{R_1}=\frac{m_2}{R_2}\;m_2=\frac{R_2}{R_1}m_1=\frac{d_2}{d_1}m_1=\frac{2{,}6}{2{,}5}24{,}096=25{,}059\,a.m.u$$

$$\frac{m_1}{R_1}=\frac{m_3}{R_3}\ m_3=\frac{R_3}{R_1}m_1=\frac{d_3}{d_1}m_1=\frac{2,7}{2,5}24,096=26,024\,a.m.u$$

Total ion detected is: 1572 + 202+226 =2000 ions

Natural abundance of $^{A1}Mg^{+2}$: $x_1=\frac{1572}{2000}\times 100=78,6$.

Natural abundance of $^{A2}Mg^{+2}$: $x_2=\frac{202}{2000}\times 100=10,1$

Natural abundance of $^{A3}Mg^{+2}$: $x_3=\frac{226}{2000}\times 100=11,3$

The atomic mass of natural magnesium: $M=\frac{\sum x_i M_i}{100}$

$$M=\frac{x_1 M_1+x_2 M_2+x_3 M_3}{100}=\frac{78,6\times 24,096+10,1\times 25,059+11,3\times 26,024}{100}=24,41\,a.m.u$$