

## Exercise Sheet N°2 D'Algebra 1

Promotion: First-Year Engineering Technology Section B

**Exercise 1** Let  $n > 0$ . Prove that if  $n$  is the square of an integer, then  $2n$  is not the square of an integer.

**Exercise 2** Prove that if  $a$  and  $b$  are two relative integers such that  $a + b\sqrt{2} = 0$ , then  $a = b = 0$ .

**Exercise 3** Consider two real numbers  $a$  and  $b$ . We consider the following proposition: if  $a + b$  is irrational, then  $a$  or  $b$  is irrational.

1. What is the contrapositive of this proposition?
2. Prove the proposition.
3. Is the converse of this proposition always true?

**Exercise 4** Show that the equation  $2x^5 - x^4 + 6x - 3$  does not admit an integer solution.

**Exercise 5** Let  $a \in \mathbb{R}$ . We consider the following proposition:

$$\forall \varepsilon > 0, |a| \leq \varepsilon \Rightarrow a = 0.$$

1. What is the contrapositive of this proposition?
2. Prove the proposition.

**Exercise 6** Show by induction that:

1.  $10^n - 1$  is a multiple of 9, for all  $n \in \mathbb{N}$ .
2.  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ , for all  $n \geq 1$ .