Series 04: Simple and multiple integrals

Exercice 01 : *Calculate using Riemann sum:* $\int_0^1 x^3 dx$ *, then determine:*

- $\sum_{k=1}^{\infty} \frac{k}{n^2+k^2}$
- $\frac{1}{n}\sum_{k=1}^{\infty} \frac{k}{\sqrt{4n^2-k^2}}$

Exercice 02 : *Using integration by parts or integration by substitution calculate:*

- $\int x^n lnx dx$,
- $\int \frac{sinx}{(cosx)^4} dx$,
- $\int arccosx dx$,
- $\int \frac{arctanx}{1+x^2} dx$.

Exercice 03 : *Calculate the double integral* $\iint_D f(x,y)dxdy$ *in the following cases:*

- $f(x,y) = \frac{x^2}{y}, \qquad D = [0,1] \times [1,2]$,
- $f(x,y) = x^2 + y^2, \qquad D = \left\{(x,y) \in \mathbb{R}^2, \quad 0 \leq x \leq 1 \ , 0 \leq y \leq x^2, \right\}$
- Using polar coordinates

$$f(x,y) = \frac{xy}{x^2+y^2}, \qquad D = \left\{(x,y) \in \mathbb{R}^2_+, \quad 1 \leq x^2+y^2 \leq 4\right\}.$$

Exercice 04 : *Calculate the triple integral:*

$$\iiint_E xyzdxdydz \qquad E = \left\{(x,y,z) \in \mathbb{R}^3, \quad 0 \leq x \leq 1 \ , x \leq y \leq 1, \ y \leq z \leq 1.\right\}$$