Series 01: Numerical series

Exercice 01 : *Tell whether the following series converges, if yes, calculate their sum:*

- $\sum_{n=1}^{\infty} \dfrac{1}{n(n+1)}$
- $\sum_{n=1}^{\infty} 2\sqrt{n} - \sqrt{n+1} - \sqrt{n-1}$

Exercice 02 : *Study the convergence of the series* $\sum_{n=1}^{\infty} u_n$ *, in the following cases:*

- $u_n = \sqrt{n^2+n} - n, \quad u_n = \left(cos\left(\dfrac{1}{n}\right)\right)^{n^2}, \quad u_n = e^{-n}, \quad u_n = 1 - cos\left(\dfrac{1}{\sqrt{n}}\right),$
- $u_n = \dfrac{2^n + n^3}{3^n + n^2}, \quad u_n = ln\left(\dfrac{2+n^\alpha}{1+n^\alpha}\right), \ \alpha \in \mathbb{R}, \quad u_n = sin\left(\frac{1}{n}\right). \quad u_n = \sqrt[n^2]{n} - 1.$

Exercice 03 : *Using the Root test (Cauchy test) and the Ratio test (D'alembert's test), study the convergence of the following series:*

- $\sum_{n=2}^{\infty} \dfrac{n}{(ln(n))^n}, \quad \sum_{n=1}^{\infty} \left(\dfrac{n}{n+1}\right)^{n^2}$
- $\sum_{n=0}^{\infty} \dfrac{3^n}{n!}, \quad \sum_{n=0}^{\infty} \dfrac{(n!)^3}{(3n!)}.$

Exercice 04 : *Let*

- $\sum_{n=1}^{\infty} (-1)^n \dfrac{ln(n)}{\sqrt{n}},$
- $\sum_{n=0}^{\infty} (-1)^n \dfrac{\sqrt{n}}{1+n}.$

Detremine whether the series is absolutely convergent, conditionally convergent or divergent.