Series 02

Exercise 01 :

1. *let* $z = \frac{e^{i\theta}}{2}$, *prove that:*

$$\frac{sin(\theta)}{2} + \frac{sin(2\theta)}{2^2} + \frac{sin(3\theta)}{2^3} + ... + ... = \frac{2sin(\theta)}{5 - 4cos(\theta)}$$

2. *Let* $z, z' \in \mathbb{C}$, *such that:*

$$|z| = |z'| = 1, \quad 1 + zz' \neq 0, \quad W = \frac{z + z'}{1 + zz'}.$$

- Show that $\bar{z} = \dfrac{1}{z}$,
- Calculate $\overline{W}$,
- What do you conclude?

Exercise 02 : *Let* $z \in \mathbb{C}$

A) Using the definition of the limit, show that $\lim_{n \to +\infty} \dfrac{n - i}{n + 1} = 1$.

B) Prove that $\lim_{z \to 0} \dfrac{\bar{z}}{z}$ does not exists.

C) Using the definition of the limit, show that $f(z) = z^2$ is continuous in $\mathbb{C}$.

D) Say whether the following functions are uniform or multiform

$$f(z) = z^3 - i\bar{z}, \quad f(z) = \sqrt[3]{z}, \quad f(z) = \sqrt{z}.$$

Exercise 03 :

1. Let $z \in \mathbb{C}$, using the definition, say whether the function $f(z) = |z|^2$ is holomorphic in $\mathbb{C}$.
2. - Prove the equivalence:

     f is holomorphic $\longleftrightarrow$ The Cauchy Riemann conditions are hold
   - What is your conclusion?