University of Badji Mokhtar-Annaba **Faculty of Engineering Sciences** **Department of Science and Technology, 2nd Year** **Pr. L. BECHIRI** **Module: Physics 3**		**2024-2025** **TD3**

Exercise 1: The equations of motion for a two-degree-of-freedom system subjected to an external force $F(t) = F_0\, cos\omega_{ex}t$ are given by

$$\ddot{x} + 10x - \theta = F_0\, cos\omega_{ex}t$$
$$\ddot{\theta} + 10\theta - x = 0$$

a) Determine the natural frequencies of the system.
b) Determine the forced oscillations of the degrees of freedom.
c) Write the general expressions for $x(t)$ *and* $\theta(t)$.

Exercise 2: A spring connecting two disks mounted on identical circular shafts is shown in Figure 1.
a) Write the equations of motion for the system using the torsion angles θ_1 and θ_2, of the shafts as coordinates for small angles θ.
b) Determine the natural frequencies of the system.
Given: $k = 5,\ C = k_t = 90,\ J = 1$ *and* $a = 2$ (in MKSA system units).

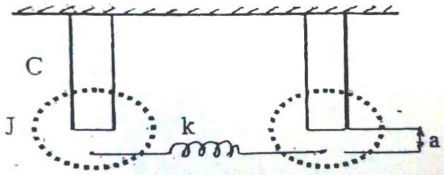

figure 1

k being the stiffness constant of the spring, $C = k_t$ the torsion constant of the shaft, and J the moment of inertia of the combined disk and shaft ensemble

Exercise 3: Consider the system shown in Figure 2.

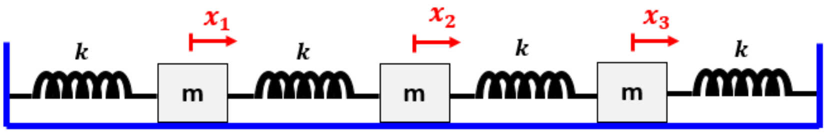

figure 2

a) Write the equations of motion for the system.
b) Determine the natural frequencies of the vibration modes of the system.
c) Write the general expressions for x_1, x_2 and x_3.
d) Assuming air friction on the body is fluid-type with a coefficient $c = 1Nms^{-1}$, determine the steady-state oscillations of the system if a force $F(t) = F_0\, sin\omega_{ex}t$ is applied to $m1$.

Exercise 4: A two-degree-of-freedom system is subjected to an external force $F(t) = F_0\, cos\omega_{ex}t$ (see figure 3).

a) Determine $x_1(t), x_2(t)$.
b) Assuming fluid-type air friction with a coefficient $c = 1Nms^{-1}$, determine the steady-state oscillations of the system.

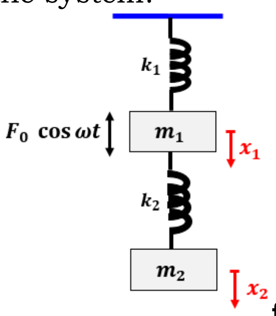

figure 3

Exercise 5: A chain of identical simple pendulums with mass m, length l, spaced by a, and coupled by springs with constant k is represented in Figure 4.

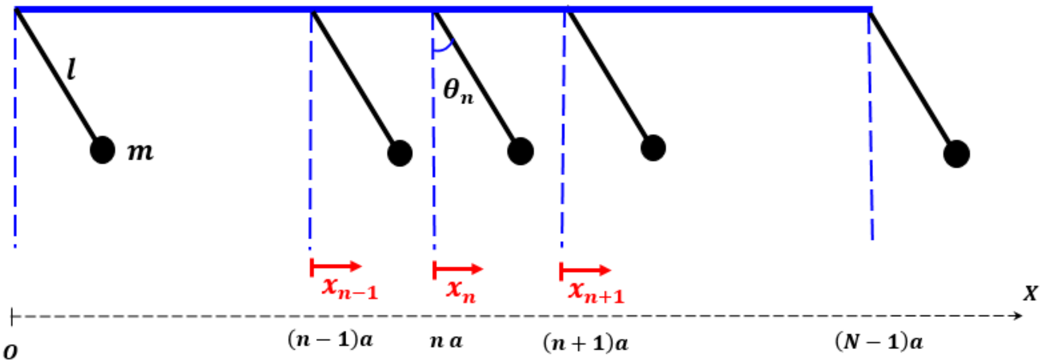

figure 4

a) Show that the equation of motion, in the vertical plane and for small angles θ, for the na x_n the pendulum bob is given by the equation:

$$\ddot{x}_n + \left(\frac{g}{l} + 2\frac{k}{m}\right)x_n - \frac{k}{m}\ (x_{n-1} + x_{n+1}) = 0$$

Where x_n, x_{n-1} and x_{n+1} are displacements along the x-axis of masses na, $(n-1)a$ *and* $(n+1)a$ respectively, g is the acceleration due to gravity.

b) In a mode of pulsation ω, regardless of boundary conditions,, x_n is given by the equation:

$$x_n = (A \sin Kna + Bcos\, Kna) \cos(\omega t + \varphi)$$

Find the dispersion relation relating ω to K.

c) In the case where the extreme pendulums ($n = 0$ *et* $n = N - 1$) are fixed, write the boundary conditions, show that the expression for x_n reduces to:

$$x_n = A \sin Kna \cos(\omega t + \varphi)$$

d) Determine the possible values of K and the maximum and minimum frequencies of the vibration modes when extreme pendulums are fixed.